One-shot Algebraic Effects as Coroutines

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Abstract. This paper presents a translation from algebraic effects and handlers to asymmetric coroutines, which provides simple, portable and widely applicable implementation of algebraic effects. Algebraic effects and handlers are an emerging new feature to model effectful computations and attract attention not only from researchers but also from programmers. They are implemented in various ways as part of compilers, interpreters, or as libraries. We present a direct embedding of one-shot algebraic effects and handlers in a language which has asymmetric coroutines. The key observation is that, by restricting the use of continuations to be *one-shot*, we obtain a simple and sufficiently general implementation via coroutines, which are available in many modern programming languages. Our translation is a macro-expressible translation, and we have implemented it embedding as a library in Lua and Ruby, which allows one to write effectful programs in a modular way using algebraic effects and handlers.

Keywords: Algebraic Effects and Handlers · Coroutines · Continuations · Control Operators · Macro Expressibility

1 Introduction

Algebraic effects [21] and handlers [22] (AEH for short) are an emerging new feature to model effectful computations in a modular way. They are gaining more and more attention not only from researchers but also from practitioners. There are a few dedicated programming languages such as Eff [1], Multicore OCaml [7] and Koka [17] which have AEH as language primitives, and several main-stream programming languages such as Haskell, OCaml, Scala, JVM bytecode and C have library implementations for AEH. However, AEH is not yet available in many other main-stream programming languages, which is a big obstacle to utilize theoretical results on AEH in real-world software. We, therefore, think that it is an important and timely issue to develop a systematic implementation method for AEH which is available in many existing programming languages.

AEH have been so far implemented in several ways such as the one based on stack manipulation, delimited-control operators [6], or free monad. Unfortunately, none of them are fully satisfactory; The implementation method based on stack manipulation is used for JVM bytecode and C [4,18], however, an implementer needs deep insight on its internal structure. It then follows that

the implementation cost is rather high, which prevents the feature from being implemented in various language systems. The implementation method via delimited-control operators is used for OCaml and Scala [3,16]. It is a systematic way to implement AEH, since it needs no knowledge on low-level features, however, only few languages have delimited-control operators as built-in primitives. The implementation method based on free monads is yet another systematic way, and used in Haskell and Scala [14,15]. While elegant, its major demerits are that it enforces a programmer to use monadic-style, and that it is often inefficient.

This paper presents a new systematic method of implementing algebraic effects and handlers which is general, available in many languages, and simple and portable, compared to the existing implementations. The key of our method is to use coroutines to embed them in programming languages. Today we see a number of programming languages which have coroutines as a built-in feature, which makes it possible to apply our implementation method in various languages with no or little cost. While coroutines are less expressive than general delimited-control operators, they are as expressive as *one-shot* delimited control-operators, a restricted control operator that is allowed to invoke a delimited continuation at most once [20]. One-shot delimited-control operators are known to be implemented more efficiently than general, multi-shot ones, thanks to the fact that no copying of continuations is necessary [5]. Hence, we face the trade-off between expressiveness and efficiency. This paper studies the one-shot variant which gives less expressive, but more performant primitives for AEH. In fact, various control effects are expressible with the one-shot variant.

We translate one-shot AEH to asymmetric coroutines. The salient feature of our translation is that it is a *macro-expressible translation* in the sense of Felleisen. Thanks to this property, we can implement AEH as a simple library, and we have created AEH libraries for Lua [4] and Ruby[5], which have been published via github. Our libraries have been used by several users, and interested users have ported our libraries to other languages[6][7].

Our main contributions in this paper are the following.

- We show an embedding of one-shot algebraic effects and handlers. We use standard asymmetric coroutines only, and no special control features are needed. Hence our embedding is applicable to various languages as long as they have asymmetric coroutines.
- Comparing to the embedding based on free monads, our method does not force programmers to use monadic style, and our embedding is more performant in many cases than the one based on free monads.
- Our embedding is defined as local and compositional translation from algebraic effects and handlers. Thanks to this property, we can implement the embedding as a library, and in fact we have done it for Lua and Ruby, which

[4] https://github.com/nymphium/eff.lua
[5] https://github.com/nymphium/ruff
[6] https://github.com/MakeNowJust/eff.js
[7] https://github.com/pandaman64/effective-rust Since Rust does not have coroutines, `Future` is used instead.

is available on GitHub. Algebraic effects and handlers is a complex system, and the implementation can be therefore error-prone. So our formalized implementation is desirable.

This paper is organized as follows. Section 2 shows typical examples using AEH and demonstrates our algebraic-effect library for Lua. Section 3 describes the embedding method by defining the translation from λ_{eff}, a language with algebraic effect handlers, to λ_{ac}, a language with asymmetric coroutines. We also show that our translation is macro expressible in the sense of Felleisen. Section 4 discusses the extension of our model definitions for implementing our libraries in Lua and Ruby, and problems in actual use. Section 5 shows the performance evaluation of our embedding by comparing ours with the embedding based on free monads. Section 6 describes related work, and Section 7 concludes.

2 Examples of *one-shot* algebraic effects

This section illustrates programming with AEH by examples. To express them, we use the programming language Lua extended with our library, which is implemented using our embedding explained in the subsequent sections.

2.1 Exception

In our view, AEH is a generalization of exceptions, which is justified by the following examples.

The function `inst` provided by our library creates, when called with zero argument, a new label for an algebraic effect, and returns it.

```
1 local DivideByZero = inst()
```

We can invoke the labeled effect by calling the function `perform` in our library.

```
1 local div = function(x, y)
2   if y == 0 then
3     return perform(DivideByZero, nil)
4   else
5     return x / y
6   end
7 end
```

This code snippet is Lua's definition for the function `div`, which takes two argument `x` and `y`. It returns the result of dividing `x` by `y` unless `y` is `0`. If `y` is `0`, it performs the effect labeled by `DivideByZero`, which means that an effect is raised and the control of the program is brought to the nearest effect handler (which is not shown in the above code) similarly to exception handling.

Our library provides the function `handler` to create a new effect handler.

```
1 local with_nil = handler {
2   val = function(_) return nil end,
3   [DivideByZero] = function(_, _)
4     return nil
5   end
6 }
```

The function `handler` receives a `table`, Lua's data structure for an associative array[8], as its sole argument in which `e1 = e2` represents the key-value pair `["e1"] = e2`. The first key-value pair (Line 2) has the key `val`, and defines a value handler which is used when no effect happens, and the second key-value pair (Lines 3 and 4) defines how the effect `DivideByZero` is processed. The value part of the key-value pair is a function in both cases. While the value handler receives one argument (which corresponds to the result of the handled expression), the effect handler receives two arguments, the first of which is the argument of the effect invocation and the second is a delimited continuation when the effect has been invoked (up to the handler invocation).

In the above snippet, the arguments are ignored, and the whole computation returns `nil` in both cases, representing simple exception capturing. By evaluating `with_nil(function() return div(3, 0) end)`, we get `nil` as the result.

We can turn the above simple exception to a *resumable* exception by changing the effect handler as follows.

```
1 local with_default_zero = handler {
2   val = function(v) return v end,
3   [DivideByZero] = function(_, k)
4     return k(0)
5   end
6 }
```

Here we changed the second case of the handler (Lines 3 and 4) so that a parameter `k` is bound to the second argument (delimited continuation), which is invoked with the argument `0`, and its value becomes the final result.

We can test the handler `with_default_zero` as follows.

```
1 with_default_zero(function()
2   local v = div(3, 0)
3   return v + 20
4 end)
```

When we execute Line 2 of this code, the effect `DivideByZero` is performed (raised) as before. Then the handler `with_default_zero` catches it, and captures the delimited continuation `local v = □; return v + 20`, which is bound to the variable `k`. (Strictly speaking, the delimited continuation should be surrounded by the handler `with_default_zero`, but we omit it here since there

[8] https://www.lua.org/manual/5.3/manual.html#3.4.9

is no effect in the continuation and its value handler is the identity function.) Then we execute `k(0)`, which is equivalent to `local v = 0; return v + 20`. The net effect is the same as the case when `div(3,0)` returns `0`, and the entire computation results in `0 + 20 = 20`.

2.2 State

AEH can express not only exceptions, but also many other effects. Here, we show how state can be expressed in terms of these operations using the state-passing technique.

We first create two effect labels.

```
1 local Get = inst()
2 local Put = inst()
```

We then define the function `run` to execute stateful computations.

```
1  local run = function(init, task)
2    local step = handler {
3      val = function(_) return function() end end,
4      [Get] = function(_, k)
5        return function(s)
6          return k(s)(s)
7        end
8      end,
9      [Put] = function(s, k)
10       return function(_)
11         return k()(s)
12       end
13     end
14   }
15
16   return step(task)(init)
17 end
```

The function takes two arguments `init` for the initial state (such as a single value or a tuple of several values) and a thunk `task` for the stateful computation. It first defines the handler `step`, which manipulates the normal-return case and the two effects labeled by `Get` and `Put`. Following the state-passing scheme, the value handler returns a function which ignores its argument (for state). In the stateful computation, when the effect `Get` is invoked, then the handler returns the function that retrieves the current state `s`. and supplies it to the current continuation (`k(s)` in line 6) with the same state `s`. When the effect `Put` with an parameter `s` is invoked, the handler returns a thunk in which a meaningless value `()` is passed to the continuation, but a new state `s` is installed (line 11). After defining the handler, the function `run` executes the computation `task` with the initial state `init` (line 16).

Note that it is important that the captured continuation is surrounded by the same handler `step`. In fact, the algebraic effects and handler are similar to the control operators shift0 and reset0 [19]; when an effect is invoked by shift0 and captured by reset0, the captured delimited continuation is surrounded by the delimiter reset0.

2.3 Expressing other Computational Effects

We can express other advanced control effects using one-shot algebraic effects and handlers. Examples include generators and iterators, let-insertion in partial evaluation, and Go language's `defer`[9], Due to lack of space, we cannot show these examples in this paper. See the github repository of our library. We have already implemented async/await, shift/reset, fetching current time (a sort of dependency injection) and measuring execution time, by our library.

3 Embedding Algebraic Effects with Coroutines

This section explains our translation from one-shot algebraic effects and handlers to asymmetric coroutines. For this purpose, we define λ_{eff}, a language which has *one-shot* AEH, and λ_{ac}, a language which has asymmetric coroutines. We then translate λ_{eff} to λ_{ac}, and show that it is a macro-expressible translation.

3.1 λ_{eff}

λ_{eff} is an untyped language with one-shot AEH based on Effy [23]. For simplicity, we omit dynamic creation of effect lablels.

Figure 1 defines the syntax of λ_{eff}. The set *Effects* is a finite set of effect lables, and we use *eff* as meta variables for it. The syntactic categories v, e, and h, resp. represent values, expressions and handler expressions, resp. The expression `perform` *eff* v invokes the effect *eff* with the argument v, and `with` v `handle` e evaluates e under the handler specified by the value v. A usual `let` binding is written as `let` $x = c_1$ `in` c_2.

The handler expression `handler` *eff* (`val` $x \to e_1$) $((y, k) \to e_2)$ creates a handler which catches the effect *eff* and returns the value of e_2 where y is bound to the argument of the effect-performing operation, and k is bound to the delimited continuation when the effect is invoked. The expression `val` $x \to e_1$ gives a value handler, namely, a handler which is used when the body of a handler returns normally (does not invoke an effect). For simplicity, λ_{eff} can handle only one effect per handler, whereas handlers in Effy can cope with multiple effects. But the latter can be simulated by our single-effect handlers, and our library actually provides the multi-effect variant; see Section 4.

The syntactic category w and the subsequent lines are used to define the semantics of λ_{eff}. The class w represents runtime values for function closures

[9] https://golang.org/ref/spec#Defer_statements

$$
\begin{array}{rl}
x & \in \textit{Variables} \\
\textit{eff} & \in \textit{Effects} \\
v & ::= x \mid h \mid \lambda x.\ e \\
e & ::= v \mid v\ v \mid \texttt{let}\ x = e\ \texttt{in}\ e \\
 & \quad \mid \texttt{perform}\ \textit{eff}\ v \mid \texttt{with}\ v\ \texttt{handle}\ e \\
h & ::= \texttt{handler}\ \textit{eff}\ \ (\texttt{val}\ \ x \to e)\ \ ((x, x) \to e) \\
 & \\
w & ::= \texttt{clos}\,(\lambda x.e, E) \mid \texttt{closh}\,(h, E) \\
F & ::= (\square\ e, E) \mid w\ \square \\
 & \quad \mid (\texttt{let}\ x = \square\ \texttt{in}\ e, E) \\
 & \quad \mid (\texttt{with}\ w\ \texttt{handle}\ \square)^{\textit{eff}} \\
 & \quad \mid (\texttt{with}\ \square\ \texttt{handle}\ e, E) \\
C & ::= e \mid w \\
E & ::= [] \mid (x = w) :: E \\
K & ::= [] \mid F :: K
\end{array}
$$

Fig. 1: Syntax and runtime representation of $\lambda_{\textit{eff}}$

($\texttt{clos}\,(\lambda x.e, E)$) and handlers ($\texttt{closh}\,(h, E)$) where E is a runtime enviroment, and F represents a *frame*, or a singular context, which means a 'one-step' fragment of a continuation. A (delimited) continuation K is a list of frames.

The call-by-value operational semantics of $\lambda_{\textit{eff}}$ is defined in the CEK-machine style [9]. We give it in Section A of the appendix of this paper, and here we informally explain the effect primitives only. The handler expression $\texttt{handler}\ \textit{eff}\ \ (\texttt{val}\ x \to e_v)\,((x, k) \to e_{ef}\ )$ creates a handler which consists of a value handler and an effect handler, and associates the effect label $\textit{eff}$ to it. The expression $\texttt{with}\ h\ \texttt{handle}\ e$ (which is called a handling expression) evaluates the expression e under the handler h. The expression $\texttt{perform}\ \textit{eff}\ v$ invokes the effect $\textit{eff}$ with an argument v. Note that handling expressions may be nested, and an effect invocation is caught (handled) by the nearest (innermost) handler which can handle the effect. When the handled expression is evaluated to a value, the value handler is used.

3.2 λ_{ac}

De Moura and Ierusalimschy's seminal work [20] classified various forms of coroutines found in programming languages, and formalized calculi for symmetric coroutines and asymmetric coroutines. The former represents classic coroutines which can call (resume) other coroutines, but coroutines cannot return to their callers. The latter represents modern coroutines where the caller-caller relation exists, hence, coroutines may return to their callers.

The language λ_{ac} is based on asymmetric coroutines[10]. For the purpose of translation and practical programming, we have added to this language several constructs such as data constructors, `let` with recursion, pattern matching, and comparison operators.

Figure 2 defines the syntax of λ_{ac}. The syntactic categories K and l, resp., represent data constructors and labels for coroutines, resp. The set *eff* corresponds to the set of effect labels in λ_{eff}, and we assume that its elements are constants in λ_{ac} Values v are either constants, an expression formed by applying a data constructor to values $K\ \overrightarrow{v}^{\star}$, labels, variables, or lambda expressions. Expressions e are those in lambda calculus extended with conditional expressions, pattern matching and mutual recursion, plus those for asymmetric coroutines: $l : e$ for a labeled expression which represents the "return point" of resuming a coroutine, `create` e for creating a coroutine and returning its label, `resume` $e_1\ e_2$ for resuming (calling) a coroutine, and `yield` e for yielding a value and returning to the caller of the current coroutine.

$f\ \overrightarrow{x}$ is an abbreviation of $f\ x_0\ x_1\ \cdots\cdots\ x_n$ and $\overrightarrow{\texttt{and}\ g\ \overrightarrow{y} = e}$ is of $\texttt{and}\ g_0\ \overrightarrow{y} = e_0\ \texttt{and}\ g_1\ \overrightarrow{y} = e_1\ \texttt{and}\ \cdots\cdots \texttt{and}\ g_m\ \overrightarrow{y} = e_m$. A similar abbreviation is applied to constructors and pattern matching.

The expression `match` e `with` *cases* is for pattern matching. We add restricted guards to pattern matching so that *cases* may contain a form $K\ \overrightarrow{x}\ \texttt{when}\ x = x \to e$. This restricted form is sufficient for our purpose.

The call-by-value operational semantics of λ_{ac} is defined in the same way as de Moura and Ierusalimschy and given in Section B of Appendix. Here we briefly explain the semantics of the primitives for coroutine; `create` e creates a unique label and a coroutine with its body being the value of e, and returns the label. The expression `resume` $l\ v$ resumes the coroutine labeled with l against the argument v. It is an error if a coroutine whose label is l does not exist, or has already been called. A resumed coroutine must return to the caller, so we create an expression $l : e_3$ where e_e is the body of the resumed coroutine. When an expression `yield` v is called in the evaluation of a coroutine, the coroutine is suspended and stored for future use, and v is returned to the caller of the current coroutine. It is an error if there is no caller of the current coroutine when `yield` is invoked.

3.3 Translation from λ_{eff} to λ_{ac}

We present a program translation from λ_{eff} to λ_{ac}, which is syntax-directed and compositional. The whole translation is defined in Figure 3 where a λ_{eff}-term e is translated to a λ_{ac}-term $[\![e]\!]$.

The translation is homomorphic for a variable, a λ-abstraction, an application, and the `let` expression. An effect label *eff* is translated to a constant with the same name.

[10] More strictly speaking, our calculus is the one for *stackful* asymmetric coroutines according to de Moura and Ierusalimschy's classification.

$$
\begin{array}{rcl}
x & \in & \mathit{Variables} \\
K & \in & \{\mathit{Eff}, \mathit{Resend}, \mathit{True}, \mathit{False}\} \\
l & \in & \mathit{Labels} \\
\mathit{eff} & \in & \mathit{Effects} \\
v & ::= & \texttt{nil} \mid \mathit{eff} \mid K\ \overrightarrow{v}^{\star} \mid l \mid x \mid \lambda x.e \\
e & ::= & v \mid K\ \overrightarrow{e}^{\star} \mid l : e \mid e\ e \mid \texttt{let}\ x = e\ \texttt{in}\ e \\
 & & \mid \texttt{match}\ e\ \texttt{with}\ \mathit{cases} \\
 & & \mid \texttt{create}\ e \mid \texttt{resume}\ e\ e \mid \texttt{yield}\ e \\
\mathit{letrec} & ::= & \texttt{let rec}\ x\ \overrightarrow{x} = e\ \left[\overrightarrow{\texttt{and}\ x\ \overrightarrow{x} = e^{\star}}\right]\ \texttt{in}\ e \\
\mathit{cases} & ::= & \overrightarrow{\mathit{pat}\ [\mathit{cond}] \rightarrow e;} \\
\mathit{cond} & ::= & \texttt{when}\ x = x \\
\mathit{pat} & ::= & K\ \overrightarrow{\mathit{pat}}^{\star} \mid x \\
C & ::= & \square \mid C\ e \mid v\ C \mid \texttt{let}\ x = C\ \texttt{in}\ \ e \mid \texttt{let}\ x = v\ \texttt{in}\ C \\
 & & \mid \texttt{match}\ C\ \texttt{with}\ \mathit{cases} \mid \texttt{let rec}\ f\ \overrightarrow{x} = e\ \texttt{in}\ C \\
 & & \mid C = e \mid \mathit{eff} = C \\
 & & \mid \texttt{let rec}\ f\ \overrightarrow{x} = e\ \overrightarrow{\texttt{and}\ f\ \overrightarrow{x} = e^{\star}}\ \texttt{in}\ C \\
 & & \mid \texttt{create}\ C \mid \texttt{resume}\ C\ e \mid \texttt{resume}\ l\ C \mid \texttt{yield}\ C \mid l : C
\end{array}
$$

Fig. 2: the syntax of λ_{ac}

We translate `perform` to `yield` based on the following observation. In the calculus for AEH, when an effect is invoked, the control is transferred to a handler corresponding to the effect, while in the calculus for coroutines, when a `yield` is called, the control is transferred to its parent coroutine. Hence we can emulate the behaviour of `perform` by `yield`. The translation wraps the arguments of `perform` with the tag *Eff* and translates them. This tag is used to determine whether the effect has been *yield*ed from the handled expression itself, or the effect has been resent (forwarded) by the handler. The handling expression `with` h `handle` e is translated to a simple application as the handler is mapped to a function.

The translation for a handler (the last case in Figure 3) is highly non trivial, and we shall explain it using an example.

Consider the program M in λ_{eff} with the effects C_1, C_2, and C_3 (Figure 4). Here we assume that our calculus is extended to have natural numbers arithmetic operations. Then M is translated to the program in Figure 5 where some variables and `let`-bindings are renamed or inlined for readability.

$$
\begin{aligned}
[\![x]\!] &= x \\
[\![\lambda x.e]\!] &= \lambda x.[\![e]\!] \\
[\![v_1\ v_2]\!] &= ([\![v_1]\!])\ ([\![v_2]\!]) \\
[\![\texttt{let}\ x = e\ \texttt{in}\ e']\!] &= \texttt{let}\ x = [\![e]\!]\ \texttt{in}\ [\![e']\!] \\
[\![\mathit{eff}]\!] &= \mathit{eff} \\
[\![\texttt{perform}\ \mathit{eff}\ v]\!] &= \texttt{yield}\ (\mathit{Eff}\ ([\![\mathit{eff}]\!])\ ([\![v]\!])) \\
[\![\texttt{with}\ h\ \texttt{handle}\ e]\!] &= [\![h]\!]\ (\lambda\_.[\![e]\!]) \\
[\![\texttt{handler}\ \mathit{eff}\ (\texttt{val}\ x \to e_v)\ ((x,k) \to e_{\mathit{eff}})]\!] &= \\
&\quad \texttt{let}\ \mathit{eff} = [\![\mathit{eff}]\!]\ \texttt{in} \\
&\quad \texttt{let}\ \mathit{vh} = \lambda x.[\![e_v]\!]\ \texttt{in} \\
&\quad \texttt{let}\ \mathit{effh} = \lambda x\ k.[\![e_{\mathit{eff}}]\!]\ \texttt{in} \\
&\quad \mathit{handler}\ \mathit{eff}\ \mathit{vh}\ \mathit{effh}
\end{aligned}
$$

where $\mathit{handler} =$

```
let rec handler eff vh effh th =
  let co = create th in
  let rec continue arg = handle (resume co arg)
  and rehandle k arg = handler eff continue effh (λ_.k arg)
  and handle r =
    match r with
    | Eff eff' v              when eff' = eff → effh v continue
    | Eff _ _                                 → yield (Resend r continue)
    | Resend (Eff eff' v) k   when eff' = eff → effh v (rehandle k)
    | Resend effv k                           → yield (Resend effv (rehandle k))
    | _                                       → vh r
  in continue nil
in handler
```

Fig. 3: Translation from λ_{eff} to λ_{ac}

The term after translation contains the function *handler* defined in Figure 3, which works as follows: `handler` makes a thunk from $(\lambda\_.\cdots\cdots)$, defines three functions *continue*, *rehandle* and *handle*, and then evaluates *continue* `nil`. *continue* passes *arg* to *co*, `resume`-s it, and passes the return value to *handle*. *handle* splits the process from the return value of `resume` according to the equivalence of tags and effect labels.

When *continue* is evaluated by passing *nil*, *Eff* C_1 10 is `yield`-ed first in the handled expression and caught by the innermost handler h_1. In this case, since it has an *Eff* tag and h_1 can handle C_1, the first pattern of *handle* matches it. *effh*

```
M = let h1 = handler C1
        (val v → v) ((x, k) → kx) in
    let h2 = handler C2
        (val v → v) ((x, k) → kx) in
    let h3 = handler C3
        (val v → v) ((x, k) → kx) in
    with h3 handle
    with h2 handle
    with h1 handle
      let a = perform (C1 10) in
      let b = perform (C1 13) in
      let c = perform (C3 17) in
      a + b + c
```

Fig. 4: Example program in λ_{eff}

```
⟦M⟧ = let h1 = let vh1 = λv. v in
              let effh1 = λx. λk. k v in
              handler C1 vh1 effh1 in
      let h2 = let vh2 = λv. v in
              let effh2 = λx. λk. k v in
              handler C2 vh2 effh2 in
      let h3 = let vh3 = λv. v in
              let effh3 = λx. λk. k v in
              handler C3 vh3 effh3 in
      h3 (λ_. h2 (λ_. h1 (λ_.
        let a = yield (Eff C1 10) in
        let a = yield (Eff C1 13) in
        let a = yield (Eff C3 17) in
        a + b + c )))
```

Fig. 5: Example after translation

is applied to the effect's argument 10 and a continuation. By passing *continue* as the continuation, the computation of a handled expression can be resumed, which is suspended at the yielded position. And since *continue* passes the return value of `resume` to *handle*, the effect can be handled by the same handler again. So a is bound to 10. When *Eff* C_1 13 is yielded in the continuation resumed by the handler, it is processed by h_1 again in the same way, and b is bound to 13.

When the effect C_3 is invoked, h_1 catches the effect first. h_1 can't handle C_3, so the second pattern of *handle* matches. The effect is sent to a handler one step outside, and the effect is processed by that handler. As with invoking an effect, an effect is re-sent to an outside handler by using a `yield`. At this time, the tag *Resend* wraps the effect and a continuation to indicate resending. Then, as in the first pattern, pass *continue* as a continuation.

The resent C_3 is captured at h_2. Since it has *Resend* tag and h_2 can't handle C_3, the fourth pattern of *handle* matches. As in the second pattern, it uses *yield* to re-send the effect to an outside handler. At this time, *rehandle* k is wrapped by *Resend* tag as a continuation. *rehandle* is a function that creates a handler that handles the thunk of the application of two given arguments. By setting *continue* to the value handler, the computation of the current handling expression can be resumed when the computation of the *rehandle* passed as a continuation is finished. *rehandle* has another role which adjusts the layers of the coroutines. In the second clause of *handle*, *handle* calls *yield*, so control is exited from one coroutine. In the third and fourth clauses, we could write $\lambda arg.handle\ (karg)$

instead of *rehandle* k, if only to manipulate the return value of the continuation. In this case, the layers of the coroutines would decrease, and eventually, we would get an error calling `yield` outside of coroutine. Therefore *rehandle* encapsulates the expression with coroutine internally and avoid to decreasing the layer of coroutines.

The effect resent again is captured by h_3. It has *Resend* tag and h_3 can handle C_3, so the third pattern of *handle* matches. Same to the fourth pattern, *rehandle* k is passed to *efffh* as a continuation. Then it returns 17 to the handled expression, and c is bound to 17.

The handled expression results in 40. Then h_1 receives it, and the fifth wild-card pattern of *handle* matches the untagged value, and the value is passed to the value handler. Same to the h_1, h_2 and h_3 receive the value and pass to the value handler. Finally the entire expression returns 40.

Although our translation looks complicated, we emphasize that our translation is compositional and local, syntax-directed, and does not rely on higher-order stores or other fancy features, but need only basic functionality of asymmetric coroutines. With this simplicity, several programmers have already ported our translation to other languages than Ruby and Lua.

3.4 Macro-expressible Translation

We claim that the translation from λ_{eff} to λ_{ac} in the previous section is simple and efficient. To support the former claim, this subsection shows that it is a macro-expressible translation in the sense of Felleisen. The latter claim will be discusses in the subsequent section.

Felleisen studied the notion of macro expressivity, which is a more fine-grained notion than most others to measure the expressive power of language primitives [8]. For instance, *call/cc* (call-with-current-continuation) can be translated away by a CPS translation to a pure lambda calculus, yet, it is not macro-expressible in pure lambda calculus since the translation is global. On the other hand, a simple let expression $\texttt{let}\ x = e_1\ \texttt{in}\ e_2$ can be locally translated by $(\lambda x.e_2)\ e_1$, therefore, it is macro-expressible in the pure lambda calculus.

While Felleisen defined the notion for the setting where a language L_1 is a proper extension of another language L_2, we want to compare the expressive power of two languages L_1 and L_2 where L_1 and L_2 are extensions of a common language L_0. To deal with this setting, we use Forster et al.'s definition for the macro-expressible translation [10], and we give its slightly simplified version here.

Definition 1 (Macro-expressible translation). *Let L_0 be a language, and L_1 and L_2, resp., be the language L_0 augmented with a set of primitives $X_1, \cdots, X_n$ and $Y_1, \cdots, Y_m$, resp. A translation ϕ from L_1 to L_2 is macro-expressible translation if and only if all of the following conditions hold.*

- *ϕ is homomorphic for the primitives in L_0. For instance, if a binary infix operator $\oplus$ is in L_0, then $\phi\,(e_1 \oplus e_2)$ is $\phi\,(e_1) \oplus \phi\,(e_2)$.*

– *ϕ maps each X_i of arity n to a syntactic expression M_i in L_2 which has n free variables $x_1, \cdots, x_n$ such that the following holds:*

$$\phi\left(X_i(e_1, \cdots, e_n)\right) = M_i\left[\phi\left(e_1\right)/x_1, \cdots, \phi\left(e_n\right)/x_n\right]$$

The expression in the right-hand side represents simultaneous substitution for the variables $x_1, \cdots, x_n$ in M_i.

To state the above definition we have made two simplifications. First, the equality in this definition should be, in general, semantic equality where we assume that each language is equipped with a certain semantics, but in this paper, we can regard it as syntactic equality. Second, we do not consider the case when X_i works as a binder such as the `let` expression[11], but we do not need to consider such cases.

It is easy to show that our translation in the previous subsection conforms the conditions for a macro-expressible translation.

Theorem 1. *Our translation in Figure 3 is a macro-expressible translation.*

Proof sketch. It is easy to check that our translation $[\![\cdot]\!]$ is homomorphic for the variable, lambda abstraction, application, let, the effect expression.

For the primitives of algebraic effects and handlers, we need to check each case. For the primitive `perform`, let M be `yield` (*Eff* x_1 x_2), then we have $[\![\texttt{perform}\ e_1\ e_2]\!] = M\,[[\![e_1]\!]/x_1,\ [\![e_2]\!]/x_2]$, and we are done. Other cases are similar. (end of proof sketch)

As we wrote above, a macro-expressible translation is rather discriminating, or sensitive to small differences between language primitives. Only local translations are macro-expressible translation. Since global translations such as a CPS translation and a state-passing translation do not qualify as macro-expressible one, state and first-class continuations are not macro-expressible in pure lambda calculus.

Put differently, if we have a macro-expressible translation for a primitive X in a language L_0, then we can implement X using the translation without changing any other primitives in L_0 This is a simple, but rather important property for our work, as it is a necessary condition to implement X as a simple library in L_0, unless we have an access to language's run-time, or reification is allowed.

4 Implementation

We have implemented AEH in Lua and Ruby based on the translation in Section 3. Since the translation is macro-expressible, we can realize our implementation as a simple library. Our implementations are compact. The Lua library is implemented in 160 lines and the core of the Ruby library is in 340 lines, even including comments for documentation generation. All our code is available via Github.

Several issues have arisen in the process of implementation we will address below.

[11] Felleisen considers the case where each argument may be bound by the construct.

Multiple Effect Handlers Our calculus λ_{eff} has the restriction that a handler can catch only one effect. However, this restriction is only for the presentation purpose, and in our actual implementation, one handler may catch multiple effects, All examples including the examples in this paper that use multiple effects per handler run without problems using our library. We also note that there is no critical performance downgrade of having multiple effects per handler.

Dynamic Effect Creation In the language λ_{eff}, we have no way to create new effect labels dynamically. Again this is due to simplicity, and we have eliminated this restriction in our implementation. The merit of allowing dynamic creation of effect instances is that a certain kind of effectful programs needs the uniqueness of effect instances, for instance, higher-order effects[16].

Conflict with Other Effects An assumption on our translation is that all effects are written via AEH. If our source program uses other effects besides AEH, it will cause a serious problem, since other effects may interfere with the internally used coroutines. For instance, if we use our library in Lua, and simultaneously use Lua's native coroutine directly, yielding a value in the source program may be accidentally caught by an internal coroutine. As consequence we must not use native coroutines with (our implementation of) AEH.

This problem can be solved as follows, thanks to the expressivity of AEH. See the following code.

```
local Yield = inst()

local yield = function(v)
  return perform(Yield, v)
end

local create = function(f)
  return { it = f, handled = false }
end

local resume = function(co, v)
  if co.handled then
    return co.it(v)
  else
    co.handled = true
    return handler({
      val = function(x) return x end,
      [Yield] = function(u, k)
        co.it = k
        return u
      end
    })(function() return co.it(v) end)
  end
```

```
24 end
```

The code above is an implementation of asymmetric coroutines by algebraic effects and handlers in Lua. The function `yield` should throw a value to `resume`, so `yield` should be an effect invocation and `resume` should be a handler. This correspondence is the inverse of the translation in Figure 3. So we define `Yield` effect (line 1) and `yield` function (line 3) as a wrapper for the invocation of the effect. The function `create` (line 7) creates a reference cell by a table. We represent a coroutine as a reference cell, which is initialized to the function `f` and the flag `handled` explained later. The handler `resume` (line 11) catches `Yield` effect with an argument and a continuation. This continuation is the rest of computation of the coroutine, so the handler stores the continuation to the cell and returns the value `u` (line 19 and 20). Since we provide a deep handler, it is not necessary to set the handler multiple times. The tag `handled` is to assert if the function is handled by the handler or not (line 12). The function `resume` checks the flag; if the flag is off, `resume` turns on the flag and runs the function with the handler. Otherwise, `resume` runs the function only.

Although we believe that the above technique may be used in other computation effects, it is left for future work to combine them with algebraic effect and handlers without big downgrade of performance.

5 Evaluation

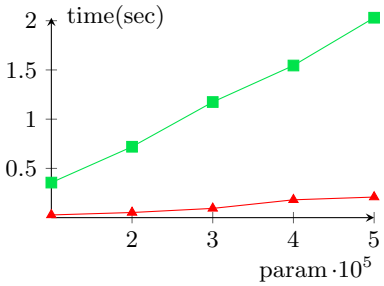
We have conducted experiments on microbenchmark using our library in Lua, and implementation in Lua based on free monads [23], and compare their performance. All the code for the benchmark is publicly available in the GitHub repository[12]. In the following figures, the symbol ▲ represents the result of our library, and ■ does of the free-monad based implementation. One of the benchmarks compares to native coroutines of Lua and indicates the result as the symbol ⋆ in a graph. The experiments have been conducted on the environment in Table 1.

Table 1: Environment for Benchmark

OS	Arch Linux
CPU	Intel Core i7-8565U
Main memory	16GB DDR4
Lua processor	LuaJIT 2.05

Figure 6 is the result of the benchmark for emulating a state monad. The benchmark uses the function `count`, cited from [14], adjusted for our library and free monad, and recursively runs a simple computation consisting of one-layer

[12] https://github.com/nymphium/effs-benchmark

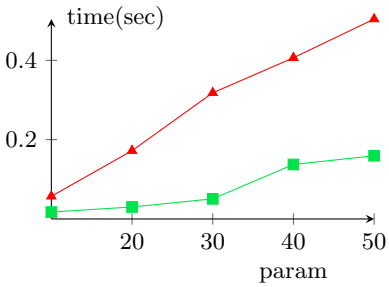


param ($10^5\times$)	1	2	3	4	5
free	0.3560	0.7199	1.1741	1.5449	2.0303
ours	0.0273	0.0521	0.0939	0.1811	0.2090

Fig. 6: Result of `onestate` benchmark

one-effect handlers for the number of times as the input parameter. The result shows that our library is approx. 10 times faster than the free-monad based implementation for this simple case. The reason why free monads are rather slow is that the bind operator requires a continuation as the next action, but the cost for creating function closures is rather high for imperative languages such as Lua. Also, functional languages such as Haskell may offer optimization for free monads, while the benchmark uses naive implementation. Nevertheless, the results are encouraging for our embedding.

In the next experiments in Figure 7, the benchmark program iterates `count` function 3,000 times in deeply nested handlers. The parameter in the table



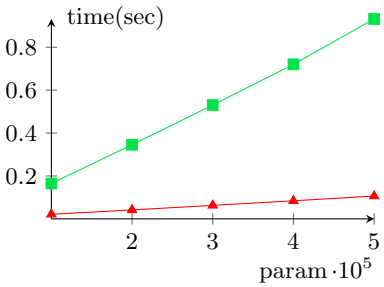
param ($10\times$)	1	2	3	4	5
free	0.0174	0.0303	0.0508	0.1374	0.1594
ours	0.0572	0.1722	0.3181	0.4064	0.5041

Fig. 7: Result of `multistate` benchmark

corresponds to the number of nested handlers/coroutines, hence 50 (the rightmost column) is already a rather unrealistic situation, but we included this experiment as an extreme. As expected, our library runs three times slower than the free monad does for this case. The reason is that *rehandle* creates a new coroutine, which is called every time an effect is caught from the other handler shown in Figure 3, so it degrades the performance.

In the next experiment, the function `looper` performs algebraic effects in the iteration of the `for` loop, where the number of iteration is given as a parameter

shown in the table of Figure 8. The benchmark program invokes an effect in a

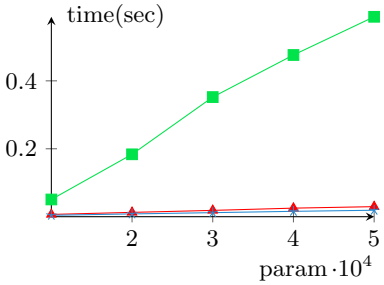


param ($10^5\times$)	1	2	3	4	5
free	0.1650	0.3455	0.5304	0.7204	0.9319
ours	0.0215	0.0416	0.0630	0.0844	0.1064

Fig. 8: Result of `looper` benchmark

`for`-loop and sets a handler out of the loop to catch the effect. Our library runs 9 times as fast as the free-monad based implementation. Note that free monads need the `forM`-operator which has large overhead. Again an advanced compiler may be able to eliminate all or part of this overhead.

Figure 9 shows the result of the benchmark, which solves the same-fringe problem [11] by using algebraic effects and coroutines. The problem is to deter-



param ($10^4\times$)	1	2	3	4	5
free	0.0507	0.1837	0.3522	0.4761	0.5886
ours	0.0067	0.0127	0.0186	0.0252	0.0296
coroutines	0.0042	0.0082	0.0119	0.0158	0.0190

Fig. 9: Result of `same_fringe` benchmark

mine whether given two trees have the same "fringe", an enumeration of leaves of the tree in a certain order. The benchmark is given the number of leaves as a parameter. We implement coroutines to solve it, by algebraic effects with free monad, and our library, described in Section 4. We also implement the solver with native coroutines of Lua. Our library yields 18 times performance gain compared to the free-monad method. Remarkably, our library is only 1.6 times slower than native coroutines.

In summary, our way of implementing AEH is advantageous in several programming languages from the performance viewpoint. We also emphasize that

writing effectful programs using coroutines is harder than writing the same programs using AEH, which provide high-level abstraction.

6 Related Work and Discussion

In this section, we discuss closely related work which has not been mentioned in this paper and picks up a few important issues for discussion.

Shallow Handler We have shown the embedding with *deep handlers*, which can catch the effect invocation even during the execution of the continuation.

In the literature, there has been a discussion on deep vs *shallow handlers* [12], and it has its own merits. We have also implemented the shallow handler with coroutines shown in Figure 10. The idea is simple; after a handler catches an

```
⟦handler† eff (val x → e_v) ((x, k) → e_eff)⟧ =
                                                  let eff = ⟦eff⟧ in
                                                  let vh = λx.⟦e_v⟧ in
                                                  let effh = λx k.⟦e_eff⟧ in
                                                  handler† eff vh effh
where handler† =
  let rec handler eff vh effh th =
    let co = create th in
    let rec continue arg = handle (resume co arg)
    and rehandle k arg = handler eff continue effh (λ_.k arg)
    and continue0 = resume co
    and rehandle0 k = resume (create k)
    and handle r =
      match r with
      | Eff eff' v              when eff' = eff → effh v continue0
      | Eff _ _                                 → yield (Resend r continue)
      | Resend (Eff eff' v) k   when eff' = eff → effh v (rehandle k)
      | Resend effv k                           → yield (Resend effv (rehandle0 k))
      | _                                       → vh r
    in continue nil
  in handler
```

Fig. 10: translation from shallow handlers to coroutines

effect, it always resends any effects to the outer handler. We have explained the role of *rehandle* in Figure 3 that it encapsulates the continuation with a coroutine to adjust the layer of coroutines, and rehandles the effect invocation in the continuation. In the shallow setting, it is also necessary to reset the number of the layer of coroutines, which might degrade the performance. On the other hand, rehandling is not needed because it is shallow.

One-shot Continuations It should be noted that we are not the first to study the one-shot variant of control operators. Bruggeman et al. give an one-shot control operator *call/1cc* with the observation that almost continuations are run at most once [5]. When a compiler knows a continuation can be run at most once, it can generate more sophisticated code. They state that, by replacing *call/cc* using continuations at most once for *call/1cc*, program can be run with less memory consumption and higher performance. Berdine et al demonstrate that many control abstractions can be translated into typed CPS including one-shot continuations, with linear-types [2].

James and Sabry stated that the *yield* operator for generator, which is a restricted variant of coroutines and can be found in various languages, is one-shot delimited continuations [13]. They also defined a generalized *yield* operator which has multi-shot continuations and show the connection between it and the delimited-control operators.

Multicore OCaml is a dialect of OCaml which natively supports algebraic effects by runtime stack manipulation. Its motivation is to write concurrent programming in direct-style[7]. They provide one-shot continuations due to the performance problem, and if multi-shot continuations are needed, they allow explicit copy for continuations.

Free monad We have already compared our with with free-monad based implementations of algebraic effects. On the positive side, it gives a systematic and elegant implementation for various effects. Its downside is it has significant overhead in performance. We also point out that our embedding-based implementation does not interfere with surface languages, while free-monad based implementations force a programmer to use monadic style, which is good for some programmers, but is not for others. With our implementation, the surface language with algebraic effects and handlers can be presented in direct style or monadic style.

7 Conclusion

We have presented a novel embedding technique for algebraic effects and handlers into asymmetric coroutines, and shown translation from the former to the latter as simple, direct, syntax-directed compositional translation. Compared with other embeddings or other ways, our technique can apply to many languages which have coroutines due to the simplistic nature of our embedding. We have demonstrated the applicability of our embedding by implementing the libraries

in Lua and Ruby. Our technique seems to be attractive for other researchers, and some of them have implemented our translation for other languages such as JavaScript and Rust. We expect that the simplicity of our implementation is advantageous to be used by more people, more languages, and more applications.

The key of our development is the one-shotness restriction of continuations. Our embedding uses the rest of the coroutine thread as a continuation, and the status of the coroutine cannot be copied, so the limitation exists that a continuation can be executed at most once. One-shotness is a dynamic property, and its static approximation, linearly used (delimited) continuations, or linear continuation-passing style, are the target of active research in the past. We hope that the formal foundation of this paper's result is studied more deeply, and coroutines and their connection with other control operators find a solid theoretical foundation.

We briefly state future work. There are many directions to extend our work. Of particular interest is to prove the semantics preservation of our translation. Introducing an appropriate type system is also an interesting next step. Another exciting issue is to relate and compare various control abstractions in the literature and in the practical programming languages. For instance, React, a popular web framework for JavaScript, has a utility software Hooks[13], which allows programmers to build components with side-effects modularly. Abramov pointed out the relevance between Hooks and algebraic effects in his blog post[14], and we think that investigating this relationship based on our work is promising.

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A Semantics of λ_{eff}

A.1 Helper functions

We introduce three helper functions for semantics in Figure 11:

$$
\begin{aligned}
\mathit{split}\left(\left((\texttt{with}\ w\ \texttt{handle}\ \square)^{\mathit{eff}} :: K\right), \mathit{eff}\right) &= \left([], (\texttt{with}\ w\ \texttt{handle}\ \square)^{\mathit{eff}}, K\right) \\
\mathit{split}\left((F :: K), \mathit{eff}\right) &= \left(F :: K', F', K''\right) \\
&\quad \text{where } F \neq (\texttt{with}\ w\ \texttt{handle}\ \square)^{\mathit{eff}} \\
&\quad\ \text{and } \left(K', F', K''\right) = \mathit{split}\left(K, \mathit{eff}\right) \\
(\!|F :: K|\!) &= \lambda x.\ (\!|K|\!)\ F\,[x] \\
(\!|\ []\ |\!) &= \lambda x.\ x \\
K * E &= \texttt{clos}\,(\lambda x.(\!|K|\!)\ x, E)
\end{aligned}
$$

Fig. 11: Helper Functions for Semantics

$\mathit{split}\,(K, \mathit{eff})$ returns a triple (K_1, F, K_2) where F is the frame that handles the effect named by *eff*, and $K = K_1 :: [F] :: K_2$ holds. If more than one frame can handle the effect *eff*, the first one is selected, and if none have the named effect, the result is undefined. $(\!|K|\!)$ converts a stack K to a continuation in functional form. $(K * E)$ creates a closure with a stack frame K and an environment E.

A.2 Small-step semantics

Figure 12 defines the small-step, call-by-value, left-to-right semantics ($\longrightarrow_{\text{eff}}$) in the CEK-machine style [9].

In the rule Lookup, $E\,(x)$ is the value associated with the variable x in the environment E. The rules PushLet, Bind, and Close, PushApp, PushArg, and App are standard. The rest of the rules are the one for algebraic effects and handlers. The rules PushWithHandle and CloseHandler push or pop evaluation contexts to the stack. The rule Handle manipulates a with-expression $\texttt{with}\ h\ \texttt{handle}\ e$: if h evaluates to a handler value, then e is going to be evaluated under this handler. The rule PushPerform pushes the frame of `perform`-ing an effect *eff* to the stack. The rules HandlePerform and HandleValue are the key rules for algebraic handlers. In the rule HandlePerform, the code component is a value w. Hence, the first frame in the stack $\texttt{perform}\ \mathit{eff}\ \square$ is retrieved and evaluated. Then we look for a handler whose name is *eff* in the stack K, and if we find it, we use the handler to cope with this effect where formal parameters y and k are bound to the value w and the delimited continuation K' under environment E. We adopt the *deep* handlers, hence the handler $(\texttt{with}\ w_h\ \texttt{handle}\ \square)^{\mathit{eff}}$ remains in the stack after this step. The rule HandleValue is used when the handled expression does not invoke an effect and returns a value w. Then the value handler $(\texttt{val}\ x \to e_v)$ is used, and the handler is eliminated from the stack after this step.

B Semantics of λ_{ac}

Auxiliary functions Figure 13 defines two auxiliary functions for pattern matching of λ_{ac}. $FV_p\,(pat)$ is the set of free variables in *pat*. $matchable\,(v, pat)$ is a predicate to assert that,given a value v and a pattern *pat*, the value matches the pattern. The operator $\oplus$ concatenates two stores and $\emptyset$ is an empty store. The function *genstore* creates a new store which consists of pairs of a variable and a value (which consists of constructors). For example, by calling *genstore* with the arguments *Resend* (*Eff* w v) u (for some values w, v and u), and a nested pattern *Resend* (*Eff* y x) k, we get a new store $\emptyset\,[y \leftarrow w, x \leftarrow v, k \leftarrow u]$.

Small-step Semantics Figure 14 shows the operational semantics of λ_{ac} by the transition ($\longrightarrow_{\mathrm{ac}}$) of the state $\langle e, \theta \rangle$, an expression e and a store θ. $dom\,(\theta)$ is the domain of θ, and $\theta\,(x)$ is a value associated with the variable x. Note that even if $\theta\,(l) = \texttt{nil}$, we include l in $dom\,(\theta)$. In those cases such as introducing a variable or a label (App, Let, LetRec, Create, Match, and MatchWhen), we identify α-equivalent terms and assume that we rename bound variables appropriately for substitution to be defined at any time. The distinctive pattern `_` is similar to a variable but generates no binding after pattern matching, so we allow `_` to be overwritten. The rules contain those for variable lookup (Lookup), function application (App), `let`, and `let rec` (Let and LetRec). The function Create is to make a new coroutine. It creates a fresh label l, binds the coroutine to l, and returns its label to the context C. The function Resume produces a labelled expression, an application $\theta\,(l)$ v with a label l. $\theta\,(l)$ v is what finds the body corresponding to the label l from θ and apply v. The created labelled expression $l : \theta\,(l)$ v expresses the computation in the coroutine labelled by l. To prevent the rest of the coroutine from being referred, the rule Resume invalidates the associated

value by setting it to `nil`. The function Yield suspends the current computation of a coroutine and returns to the parent coroutine with an argument. Since the target calculus represents asymmetric coroutines, a coroutine can be a parent of another coroutine by resuming it. The rule have an assumption that C_2 does not have any labelled expresses. The assumption indicates that C_2 is the innermost active coroutine. The function LabelledReturn transfers the result of the computation v in the coroutine l to its caller. The functions EqT and EqF compare two effect operations. The operator $=_{\mathit{eff}}$ judges whether two given effects are the same. The functions Match and MatchWhen are for pattern-matching. The second rule applies when $K\ \overrightarrow{v}$ matches a pattern, and the match case has a guard c. This rule transforms the guard to another match expression, with assigning the values to the corresponding variables in the pattern. The assignment may affect pattern variables in the guard c. If a guard returns `True`, pattern matching is successful, and the body of the `True` clause is evaluated; otherwise, we go to match against the rest of the patterns.

$$\boxed{\langle C;\, E;\, K\rangle \longrightarrow_{\text{eff}} \langle C';\, E';\, K'\rangle}$$

$$\langle x;\, E;\, K\rangle \longrightarrow_{\text{eff}} \langle E\,(x)\,;\, E;\, K\rangle \qquad \text{(Lookup)}$$

$$\langle \texttt{let}\ x = e\ \texttt{in}\ e';\, E;\, K\rangle \longrightarrow_{\text{eff}} \langle e;\, E;\, (\texttt{let}\ x = \Box\ \texttt{in}\ e', E) :: K\rangle \qquad \text{(PushLet)}$$

$$\langle w;\, E;\, (\texttt{let}\ x = \Box\ \texttt{in}\ e, E') :: K\rangle \longrightarrow_{\text{eff}} \langle e;\, (x = w) :: E';\, K\rangle \qquad \text{(Bind)}$$

$$\langle \lambda x.\ e;\, E;\, K\rangle \longrightarrow_{\text{eff}} \langle \texttt{clos}\,(\lambda x.e, E)\,;\, E;\, K\rangle \qquad \text{(Close)}$$

$$\langle e\ e';\, E;\, K\rangle \longrightarrow_{\text{eff}} \langle e;\, E;\, (\Box\ e', E) :: K\rangle \qquad \text{(PushApp)}$$

$$\langle w;\, E;\, (\Box\ e, E') :: K\rangle \longrightarrow_{\text{eff}} \langle e;\, E';\, (w\ \Box) :: K\rangle \qquad \text{(PushArg)}$$

$$\left\langle w;\, E;\, \left(\texttt{clos}\,(\lambda x.e, E)'\ \Box\right) :: K\right\rangle \longrightarrow_{\text{eff}} \langle e;\, (x = w) :: E';\, K\rangle \qquad \text{(App)}$$

$$\langle \texttt{with}\ h\ \texttt{handle}\ e;\, E;\, K\rangle \longrightarrow_{\text{eff}} \langle h;\, E;\, (\texttt{with}\ \Box\ \texttt{handle}\ e, E) :: K\rangle \qquad \text{(PushWithHandle)}$$

$$\langle h;\, E;\, K\rangle \longrightarrow_{\text{eff}} \langle \texttt{closh}\,(h, E)\,;\, E;\, K\rangle \quad \text{where } h = \texttt{handler}\ \mathit{eff}\ \ (\texttt{val}\ \ x \to e_v)\ \ ((x, k) \to e_{\mathit{eff}}) \qquad \text{(CloseHandler)}$$

$$\left\langle \begin{array}{c} w_h; \\ E'; \\ (\texttt{with}\ \Box\ \texttt{handle}\ e, E) :: K \end{array} \right\rangle \longrightarrow_{\text{eff}} \left\langle \begin{array}{c} e; \\ E; \\ \left((\texttt{with}\ w_h\ \texttt{handle}\ \Box)^{\mathit{eff}}\right) :: K \end{array} \right\rangle \quad \text{where } w_h = \texttt{closh}\,(\texttt{handler}\ \mathit{eff}\ (\texttt{val}\ x \to e_v)\ \ ((x, k) \to e_{\mathit{eff}})\,, E) \qquad \text{(Handle)}$$

$$\langle \texttt{perform}\ \mathit{eff}\ v;\, E;\, K\rangle \longrightarrow_{\text{eff}} \langle v;\, E;\, (\texttt{perform}\ \mathit{eff}\ \Box) :: K\rangle \qquad \text{(PushPerform)}$$

$$\frac{\mathit{split}\,(K, \mathit{eff}\ ) = \left(K', (\texttt{with}\ w_h\ \texttt{handle}\ \Box)^{\mathit{eff}}\,, K''\right) \quad \text{where } w_h = \texttt{closh}\,(\texttt{handler}\ \mathit{eff}\ \ (\texttt{val}\ x \to e_v)\ \ ((y, k) \to e_{\mathit{eff}})\,, E')}{\langle w;\, E;\, (\texttt{perform}\ \mathit{eff}\ \Box) :: K\rangle \longrightarrow_{\text{eff}} \left\langle \begin{array}{c} e_{\mathit{eff}}; \\ (y = w) :: (k = K' * E) :: E'; \\ (\texttt{with}\ w_h\ \texttt{handle}\ \Box)^{\mathit{eff}} :: K'' \end{array} \right\rangle} \qquad \text{(HandlePerform)}$$

$$\frac{F = (\texttt{with}\ w_h\ \texttt{handle}\ \Box)^{\mathit{eff}} \quad \text{where } w_h = \texttt{closh}\,(\texttt{handler}\ \mathit{eff}\ \ (\texttt{val}\ x \to e_v)\ \ ((y, k) \to e_{\mathit{eff}})\,, E')}{\langle w;\, E;\, F :: K\rangle \longrightarrow_{\text{eff}} \langle e_v;\, (x = w) :: E';\, K\rangle} \qquad \text{(HandleValue)}$$

Fig. 12: Semantics of λ_{eff}

$$FV_p\left(K\ \overrightarrow{pat}\right) = \bigcup p \in \overrightarrow{pat}.FV_p\left(p\right)$$
$$FV_p\left(x\right) = \{x\}$$

$$matchable\left(K\ \overrightarrow{v}, K'\ \overrightarrow{pat}\right) = K =_K K' \wedge \forall v \in \overrightarrow{v}, p \in \overrightarrow{pat}.matchable\left(v, p\right)$$

$$\theta_1 \oplus \theta_2 = \emptyset \begin{bmatrix} \forall x \in dom\left(\theta_1\right).x \leftarrow \theta_1\left(x\right), \\ \forall y \in dom\left(\theta_2\right).y \leftarrow \theta_2\left(y\right) \end{bmatrix}$$

$$genstore\left(K\ \overrightarrow{v}, K\ \overrightarrow{pat}\right) = \bigoplus_{v \in \overrightarrow{v}, p \in \overrightarrow{pat}} genstore\left(v, p\right)$$
$$genstore\left(v, x\right) = \emptyset\left[x \leftarrow v\right]$$

Fig. 13: Auxiliary functions for the semantics of λ_{ac}

$$\langle C[x], \theta\rangle \longrightarrow_{\mathrm{ac}} \langle C[\theta(x)], \theta\rangle \qquad \text{(Lookup)}$$

$$\frac{x \notin dom\,(\theta)}{\langle C\,[(\lambda x.e)\,v]\,, \theta\rangle \longrightarrow_{\mathrm{ac}} \langle C\,[e]\,, \theta\,[x \leftarrow v]\rangle} \qquad \text{(App)}$$

$$\frac{x \notin dom\,(\theta)}{\langle C[\texttt{let}\ x = v\ \texttt{in}\ e'], \theta\rangle \longrightarrow_{\mathrm{ac}} \langle C[e], \theta[x \leftarrow v]\rangle} \qquad \text{(Let)}$$

$$\frac{\forall z \in \left\{f, \overrightarrow{x}, g, \overrightarrow{\overrightarrow{y}}\right\}.z \notin dom\,(\theta)}{\left\langle C\left[\begin{array}{l}\texttt{let rec}\ f\ \overrightarrow{x} = e_f \\ \overrightarrow{\texttt{and}\ g\ \overrightarrow{y} = e_g} \\ \texttt{in}\ e\end{array}\right], \theta\right\rangle \longrightarrow_{\mathrm{ac}} \left\langle C[e], \theta\left[\begin{array}{l}f \leftarrow \lambda\overrightarrow{x}.e_f, \\ \overrightarrow{g \leftarrow \lambda\overrightarrow{y}.e_g}\end{array}\right]\right\rangle} \qquad \text{(LetRec)}$$

$$\frac{l \notin dom\,(\theta)}{\langle C\,[\texttt{create}\ v]\,, \theta\rangle \longrightarrow_{\mathrm{ac}} \langle C\,[l]\,, \theta\,[l \leftarrow v]\rangle} \qquad \text{(Create)}$$

$$\langle C\,[\texttt{resume}\ l\ v]\,, \theta\rangle \longrightarrow_{\mathrm{ac}} \langle C\,[l : \theta\,(l)\ \ v]\,, \theta\,[l \leftarrow \texttt{nil}]\rangle \qquad \text{(Resume)}$$

$$\frac{C_2 \text{ does not contains labelled expressions}}{\langle C_1\,[l : C_2\,[\texttt{yield}\ \ v]]\,, \theta\rangle \longrightarrow_{\mathrm{ac}} \langle C_1\,[v]\,, \theta\,[l \leftarrow \lambda x.C_2\,[x]]\rangle} \qquad \text{(Yield)}$$

$$\langle C\,[l : v]\,, \theta\rangle \longrightarrow_{\mathrm{ac}} \langle C\,[v]\,, \theta\rangle \qquad \text{(LabelledReturn)}$$

$$\frac{\mathit{eff} =_{\mathit{eff}} \mathit{eff}'}{\langle C\,[\mathit{eff} = \mathit{eff}']\,, \theta\rangle \longrightarrow_{\mathrm{ac}} \langle C\,[\mathit{True}]\,, \theta\rangle} \qquad \text{(EqT)}$$

$$\frac{\mathit{eff} \neq_{\mathit{eff}} \mathit{eff}'}{\langle C\,[\mathit{eff} = \mathit{eff}']\,, \theta\rangle \longrightarrow_{\mathrm{ac}} \langle C\,[\mathit{False}]\,, \theta\rangle} \qquad \text{(EqF)}$$

$$\frac{\neg \mathit{matchable}\,(K\ \overrightarrow{v}, \mathit{pat})}{\left\langle C\left[\begin{array}{l}\texttt{match}\ K\ \overrightarrow{v}\ \texttt{with} \\ \mathit{pat}\ \ [\mathit{cond}] \rightarrow e; \\ \mathit{cases}\end{array}\right], \theta\right\rangle \longrightarrow_{\mathrm{ac}} \langle C\,[\texttt{match}\ K\ \overrightarrow{v}\ \texttt{with}\ \mathit{cases}]\,, \theta\rangle} \qquad \text{(MatchNext)}$$

$$\frac{\begin{array}{c}\forall x \in FV_p\,(\mathit{pat})\,.x \notin \mathrm{dom}\,(\theta) \qquad \mathit{matchable}\,(K\ \overrightarrow{v}, \mathit{pat}) \\ \theta' = \theta \oplus \mathit{genstore}\,(K\ \overrightarrow{v}, \mathit{pat})\end{array}}{\langle C\,[\texttt{match}\ K\ \overrightarrow{v}\ \texttt{with}\ \mathit{pat} \rightarrow e; \mathit{cases}]\,, \theta\rangle \longrightarrow_{\mathrm{ac}} \langle C[e], \theta'\rangle} \qquad \text{(Match)}$$

$$\frac{\begin{array}{c}\forall x \in FV_p\,(\mathit{pat})\,.x \notin \mathrm{dom}\,(\theta) \qquad \mathit{matchable}\,(K\ \overrightarrow{v}, \mathit{pat}) \\ \theta' = \theta \oplus \mathit{genstore}\,(K\ \overrightarrow{v}, \mathit{pat})\end{array}}{\left\langle C\left[\begin{array}{l}\texttt{match}\ K\ \overrightarrow{v}\ \texttt{with} \\ \mathit{pat}\ \texttt{when}\ c \rightarrow e; \\ \mathit{cases}\end{array}\right], \theta\right\rangle \longrightarrow_{\mathrm{ac}} \left\langle C\left[\begin{array}{l}\texttt{match}\ c\ \texttt{with} \\ \mathit{True} \rightarrow e; \\ \mathit{False} \rightarrow \\ \quad \texttt{match}\ K\ \overrightarrow{v}\ \texttt{with} \\ \quad \mathit{cases}\end{array}\right], \theta'\right\rangle} \qquad \text{(MatchWhen)}$$

Fig. 14: Semantics of λ_{ac}