『プログラム言語論』 演習課題 (5/23 出題分) の解答例

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1 宿題

L_2 に対応する CEK 機械で、以下のものの状態遷移を書きなさい。(計算は $\langle eval, e, \texttt{[]}, \texttt{init}\rangle$ という状態からはじめる。また、ステップ数が非常に多い場合は、多少省略して書いてもよい。)

- e が $\texttt{let}\ x = 1\ \texttt{in}\ x + 2 * 3$(具体構文での記述) のとき。

  答は以下の通り (見やすさのため、抽象構文でなく具体構文で表記している):

  [2013/6/20; 昨日までここに置いていた解答ファイルでは、与えられた式の括弧付けを間違えていたので、修正しました。Thanks to 須永君]

$$
\begin{aligned}
&\langle eval, \texttt{let}\ x = 1\ \texttt{in}\ x + 2 * 3, \texttt{[]}, \texttt{init}\rangle \\
&\to \langle eval, 1, \texttt{[]}, \texttt{push}((letin, x, x + 2 * 3, \texttt{[]}), \texttt{init})\rangle \\
&\to \langle apply, \texttt{push}((letin, x, x + 2 * 3, \texttt{[]}), \texttt{init}), 1\rangle \\
&\to \langle eval, x + 2 * 3, \texttt{[]}[x = 1], \texttt{init}\rangle \\
&\to \langle eval, x, \texttt{[]}[x = 1], \texttt{push}((plus1, 2 * 3, \texttt{[]}[x = 1]), \texttt{init})\rangle \\
&\to \langle apply, \texttt{push}((plus1, 2 * 3, \texttt{[]}[x = 1]), \texttt{init}), 1\rangle \\
&\to \langle eval, 2 * 3, \texttt{[]}[x = 1], \texttt{push}((plus2, 1), \texttt{init})\rangle \\
&\to \langle eval, 2, \texttt{[]}[x = 1], \texttt{push}((times1, 3, \texttt{[]}[x = 1]), \texttt{push}((plus2, 1), \texttt{init}))\rangle \\
&\to \langle apply, \texttt{push}((times1, 3, \texttt{[]}[x = 1]), \texttt{push}((plus2, 1), \texttt{init})), 2\rangle \\
&\to \langle eval, 3, \texttt{[]}[x = 1], \texttt{push}((times2, 2), \texttt{push}((plus2, 1), \texttt{init}))\rangle \\
&\to \langle apply, \texttt{push}((times2, 2), \texttt{push}((plus2, 1), \texttt{init})), 3\rangle \\
&\to \langle apply, \texttt{push}((plus2, 1), \texttt{init}), 6\rangle \\
&\to \langle apply, \texttt{init}, 7\rangle \\
&\to 7
\end{aligned}
$$

- e が $(\texttt{let}\ x = 1\ \texttt{in}\ \texttt{let}\ y = x + 4\ \texttt{in}\ x * y) * 3$(具体構文での記述) のとき。

  答は以下の通り (適宜省略。また見やすさのため、抽象構文でなく具体構文で表記している):

$\langle eval, (\texttt{let}\ x = 1\ \texttt{in let}\ y = x + 4\ \texttt{in}\ x * y) * 3, \texttt{[]}, \texttt{init}\rangle$

$\to \langle eval, \texttt{let}\ x = 1\ \texttt{in let}\ y = x + 4\ \texttt{in}\ x * y, \texttt{[]}, \texttt{push}((times1, 3, \texttt{[]}), \texttt{init})\rangle$

$\to \langle eval, 1, \texttt{[]}, \texttt{push}((letin, x, \texttt{let}\ y = x + 4\ \texttt{in}\ x * y, \texttt{[]}), \texttt{push}((times1, 3, \texttt{[]}), \texttt{init}))\rangle$

$\to \langle apply, \texttt{push}((letin, x, \texttt{let}\ y = x + 4\ \texttt{in}\ x * y, \texttt{[]}), \texttt{push}((times1, 3, \texttt{[]}), \texttt{init})), 1\rangle$

$\to \langle eval, \texttt{let}\ y = x + 4\ \texttt{in}\ x * y, \texttt{[]}[x = 1], \texttt{push}((times1, 3, \texttt{[]}), \texttt{init})\rangle$

$\to \langle eval, x + 4, \texttt{[]}[x = 1], \texttt{push}((letin, y, x * y, \texttt{[]}[x = 1]), \texttt{push}((times1, 3, \texttt{[]}), \texttt{init}))\rangle$

$\to \cdots \to \langle apply, \texttt{push}((letin, y, x * y, \texttt{[]}[x = 1]), \texttt{push}((times1, 3, \texttt{[]}), \texttt{init})), 5\rangle$

$\to \langle eval, x * y, \texttt{[]}[x = 1][y = 5], \texttt{push}((times1, 3, \texttt{[]}), \texttt{init})\rangle$

$\to \langle eval, x, \texttt{[]}[x = 1][y = 5], \texttt{push}((times1, y, \texttt{[]}[x = 1][y = 5]), \texttt{push}((times1, 3, \texttt{[]}), \texttt{init}))\rangle$

$\to \langle apply, \texttt{push}((times1, y, \texttt{[]}[x = 1][y = 5]), \texttt{push}((times1, 3, \texttt{[]}), \texttt{init})), 1\rangle$

$\to \langle eval, y, \texttt{[]}[x = 1][y = 5], \texttt{push}((times2, 1), \texttt{push}((times1, 3, \texttt{[]}), \texttt{init}))\rangle$

$\to \langle apply, \texttt{push}((times2, 1), \texttt{push}((times1, 3, \texttt{[]}), \texttt{init})), 5\rangle$

$\to \cdots \to \langle apply, \texttt{push}((times1, 3, \texttt{[]}), \texttt{init}), 5\rangle \to 15$

- e が $(\texttt{let}\ x = 1\ \texttt{in let}\ x = x + 4\ \texttt{in}\ x * 3) * 3$(具体構文での記述) のとき。

$\langle eval, (\texttt{let}\ x = 1\ \texttt{in let}\ x = x + 4\ \texttt{in}\ x * 3) * 3, \texttt{[]}, \texttt{init}\rangle$

$\to \cdots \to \langle eval, x, \texttt{[]}[x = 1][x = 5], \texttt{push}((times1, 3, \texttt{[]}[x = 1][x = 5]), \texttt{push}((times1, 3, \texttt{[]}), \texttt{init}))\rangle$

$\to \langle apply, \texttt{push}((times1, 3, \texttt{[]}[x = 1][x = 5]), \texttt{push}((times1, 3, \texttt{[]}), \texttt{init})), 5\rangle$

$\to \cdots \to \langle apply, \texttt{push}((times1, 3, \texttt{[]}), \texttt{init}), 15\rangle$

$\to \cdots \to \langle apply, \texttt{init}, 45\rangle \to 45$